



In[36]:= **(\* Benefactors? Donations are welcome at Wittenberg University in Springfield Ohio. Payable to the Saint Augustine Scholars Foundation. A 501(c)(3) charitable foundation. Donations are of course, tax deductible. Thanks, Alan White President, Armillary Equations, LLC \*)**

**LogisticIncrementE1L :=  
N0 + (Pf × (I0 / (I0 + (100.0 - I0) × (Exp[-(k1 × (t - Pk))]))));**

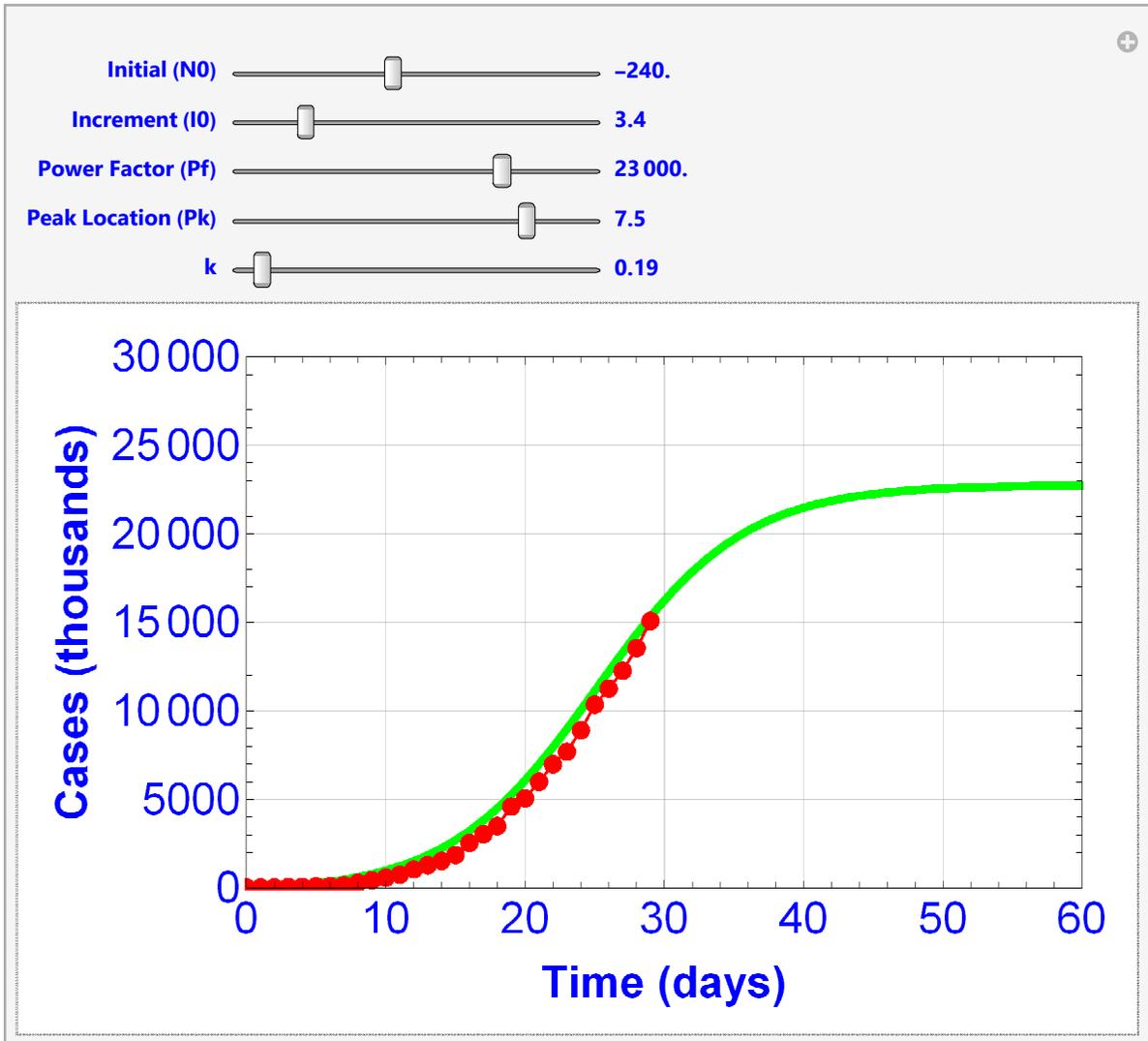
**(\* E1L from: Rubinow SI (1975) Introduction to Mathematical Biology John Wiley and Sons, New York, London ISBN: 0-471-74446-8 Pages: 11 - 17 \*)**

**Clear[t, E1Logistic, N0, Pf, I0, Pk, k]  
E1Logistic[t\_, N0\_, Pf\_, I0\_, Pk\_, k\_] :=  
N0 + (Pf × (I0 / (I0 + (100.0 - I0) × (Exp[-(k × (t - Pk))]))))**

```

In[39]:= Manipulate[Show[Plot[E1Logistic[t, N0, Pf, I0, Pk, k], {t, 0, 60},
  Frame → True, GridLines → Automatic, AxesOrigin → {0, 0},
  PlotRange → {{0, 60}, {0, 30 000}}, ImageSize → {575, 366},
  PlotStyle → {RGBColor[0, 1, 0], Thickness[0.01]},
  AspectRatio → 2.0 /  $\pi$ , FrameLabel → {Style["Time (days)",
    Bold, Blue], Style["Cases (thousands)", Bold, Blue]},
  LabelStyle → Directive[Blue, Large]], dtplt1a],
  {{N0, -240.0, Style["Initial (N0)", Blue, Bold, Medium]}, -500.0,
  100.0, 0.01, ControlType → Slider, ImageSize → Medium,
  DefaultBaseStyle → {Blue, Bold, Medium}, Appearance → "Labeled"},
  {{I0, 3.4, Style["Increment (I0)", Blue, Bold, Medium]}, 0.01,
  20.0, 0.001, ControlType → Slider, ImageSize → Medium,
  DefaultBaseStyle → {Blue, Bold, Medium}, Appearance → "Labeled"},
  {{Pf, 23 000.0, Style["Power Factor (Pf)", Blue, Bold, Medium]},
  1000.0, 30 000.0, 0.001, ControlType → Slider,
  ImageSize → Medium, DefaultBaseStyle → {Blue, Bold, Medium},
  Appearance → "Labeled"},
  {{Pk, 7.5, Style["Peak Location (Pk)", Blue, Bold, Medium]}, -5.0,
  10.0, 0.0010, ControlType → Slider, ImageSize → Medium,
  DefaultBaseStyle → {Blue, Bold, Medium}, Appearance → "Labeled"},
  {{k, 0.19, Style["k", Blue, Bold, Medium]}, 0.10, 2.5,
  0.00001, ControlType → Slider, ImageSize → Medium,
  DefaultBaseStyle → {Blue, Bold, Medium}, Appearance → "Labeled"},
  Paneled → True, ImageMargins → 0]

```



```
In[40]:= KinetE1L = NonlinearModelFit[dtpts1a, E1Logistic[t, N0, Pf, I0, Pk, k],
  {{N0, -240}, {Pf, 23850}, {I0, 0.92}, {Pk, 2.86}, {k, 0.199}},
  t, Method -> "Automatic"]
```

```
Out[40]= FittedModel[
$$-240.53 + \frac{22795.8}{0.9557 + 99.0443 e^{-\langle\langle 20 \rangle\rangle (\langle\langle 1 \rangle\rangle)}}$$

```

```
In[41]:= Normal[KinetE1L]
```

```
Out[41]= 
$$-240.53 + \frac{22795.8}{0.9557 + 99.0443 e^{-0.199604 (-3.05331+t)}}$$

```

```
In[42]:= KinetE1L["BestFitParameters"]
```

```
Out[42]= {N0 -> -240.53, Pf -> 23852.5, I0 -> 0.9557, Pk -> 3.05331, k -> 0.199604}
```

In[43]:= **KinetE1L["ParameterErrors"]**

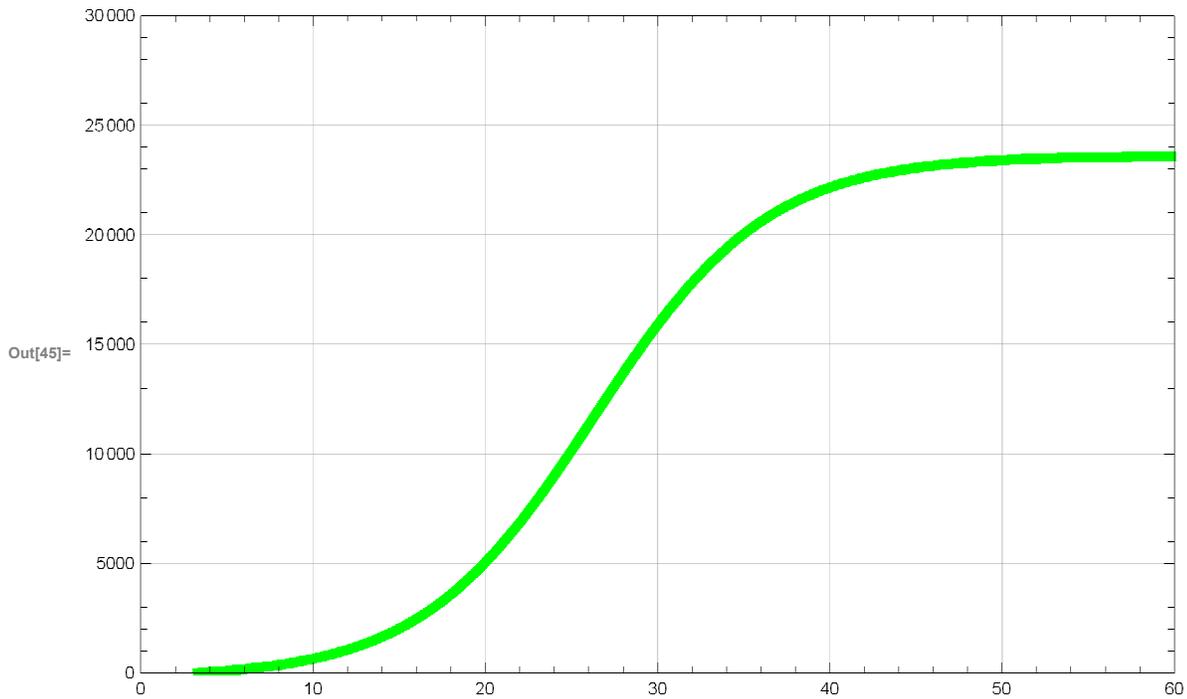
Out[43]= {69.0077, 1226.25, 0.0774334, 0.0146302, 0.00728396}

In[44]:= **KinetE1L["ParameterTable"]**

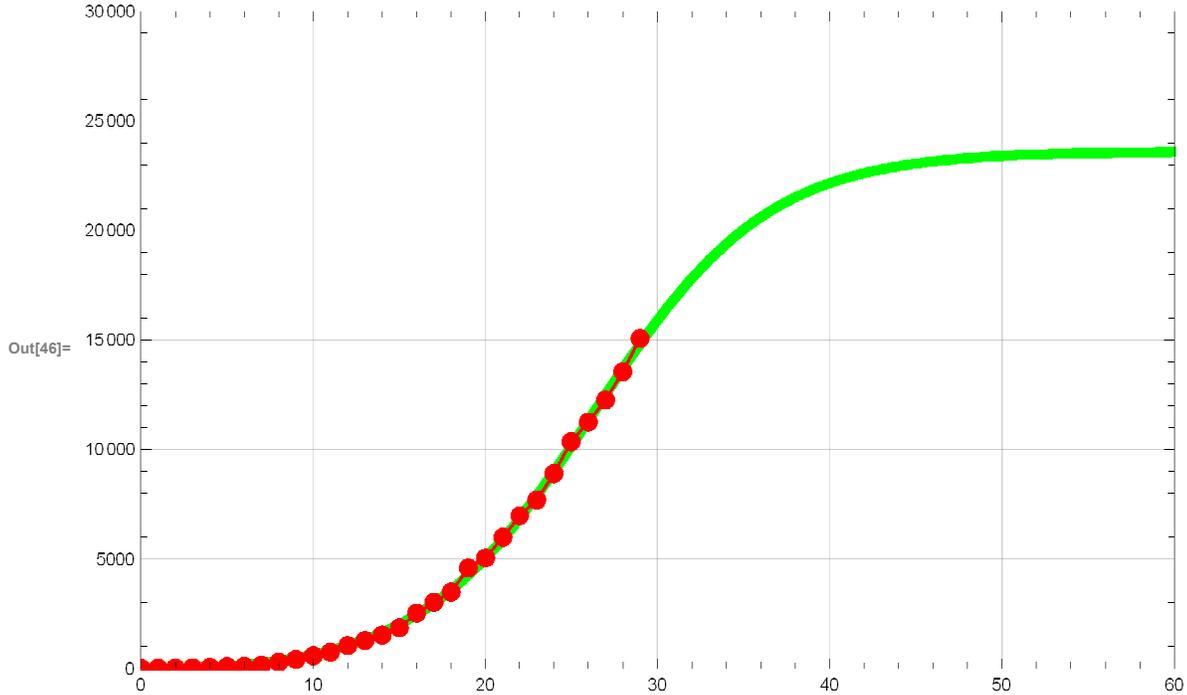
	Estimate	Standard Error	t-Statistic	P-Value
N0	-240.53	69.0077	-3.48555	0.00183061
Pf	23852.5	1226.25	19.4515	$1.30177 \times 10^{-16}$
I0	0.9557	0.0774334	12.3422	$3.92185 \times 10^{-12}$
Pk	3.05331	0.0146302	208.7	$4.8066 \times 10^{-42}$
k	0.199604	0.00728396	27.4033	$3.60255 \times 10^{-20}$

In[45]:= **KinetE1LMdl =**

**Plot[{KinetE1L[t]}, {t, 0, 60}, PlotRange → {{0, 60}, {0, 30 000}},  
 Frame → True, GridLines → Automatic, ImageSize → {575, 366},  
 PlotStyle → {Green, Thickness[0.01]}, AspectRatio → 2 ÷ π]**



In[46]:= **Show[KinetE1LMdl, dtp1t1a]**



In[47]:= **FindMaximum[KinetE1L[t], {t, 100}]**

Out[47]= {23612., {t → 1110.}}

In[48]:=

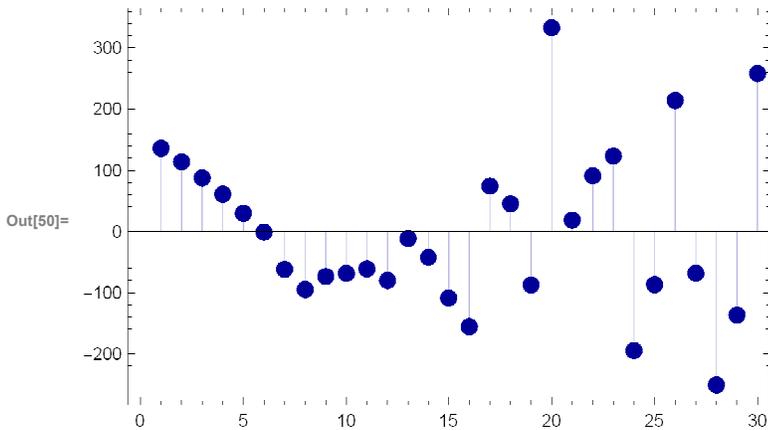
**FindMinimum[KinetE1L[t], {t, -100}]**

Out[48]= {-240.53, {t → -1110.}}

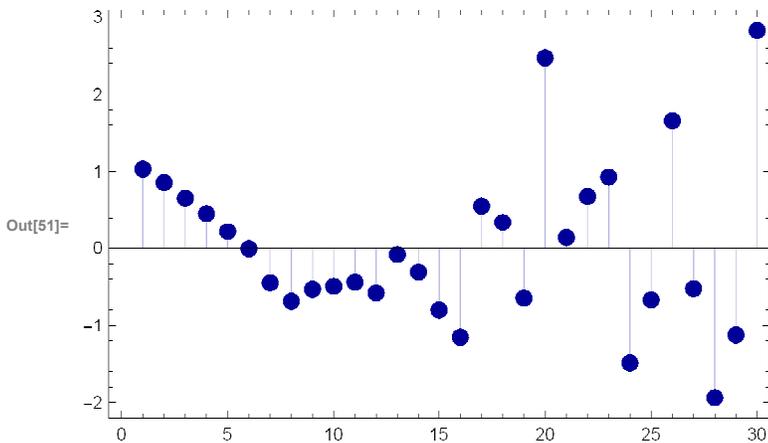
In[49]:= **residls = KinetE1L["FitResiduals"]**

Out[49]= {136.058, 113.735, 87.4605, 60.9617, 29.7034, -1.1592, -61.8358,  
 -94.9614, -73.6615, -68.6195, -61.1403, -80.2053, -11.5057, -42.4406,  
 -109.06, -155.93, 74.1148, 45.341, -87.349, 333.267, 18.5461, 90.8642,  
 123.447, -194.998, -87.0167, 214.335, -68.3438, -251.167, -136.694, 258.255}

```
In[50]:= ListPlot[residls, Frame → True, Filling → Axis,
PlotStyle → {RGBColor[0, 0, 0.6], AbsolutePointSize[9]}
```



```
In[51]:= ListPlot[KinetE1L["StandardizedResiduals"], Frame → True,
Filling → Axis, PlotStyle → {RGBColor[0, 0, 0.6], AbsolutePointSize[9]}
```

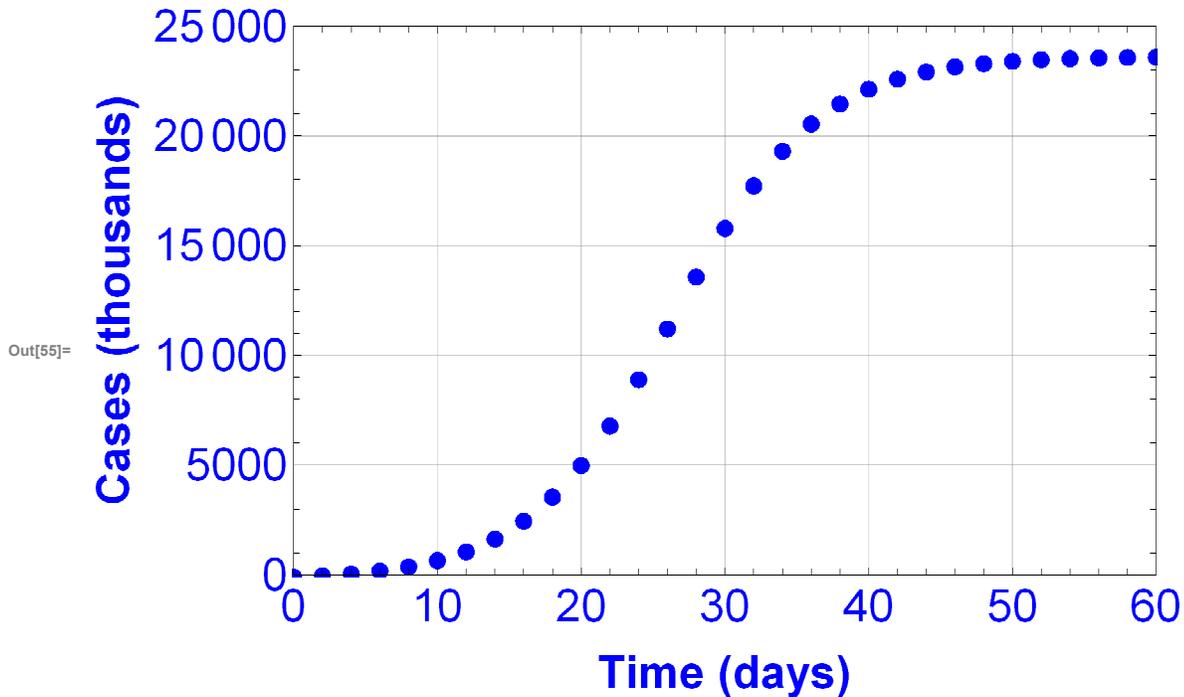


```
In[52]:= LogisticIncrementE1 :=
N0 + (Pf × (I0 / (I0 + (100.0 - I0) × (Exp[-(k1 × (t - Pk)])))));
```

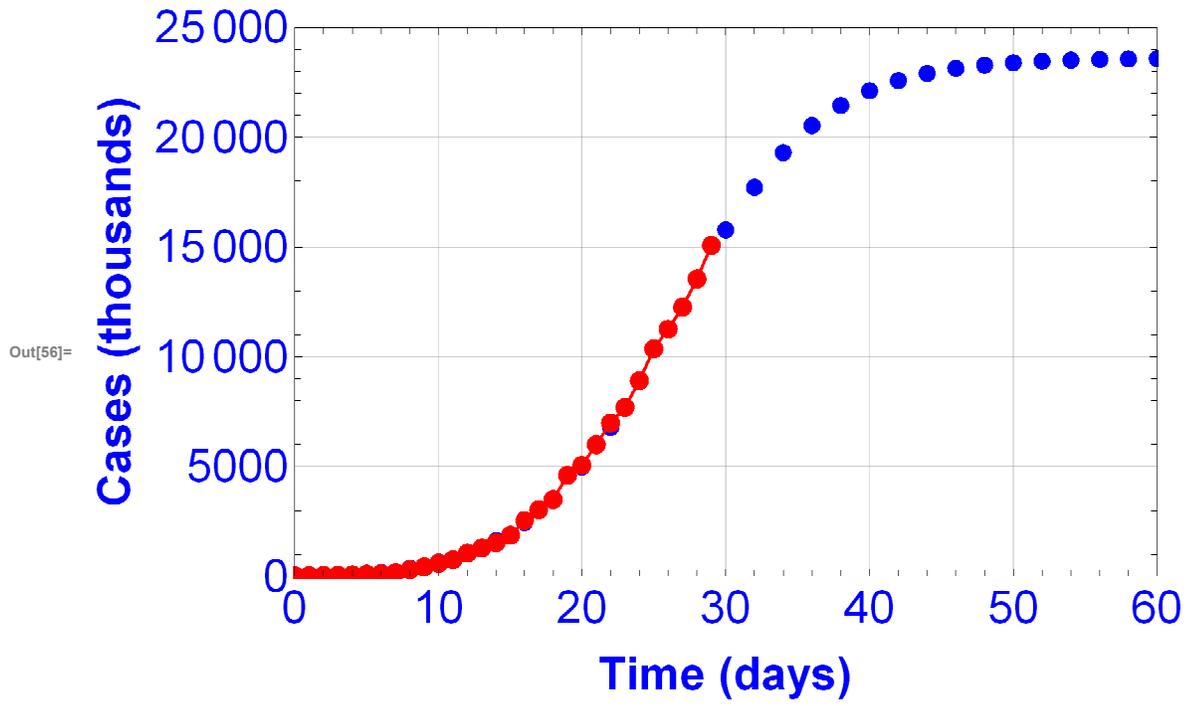
```
In[53]:= E1LMdl = test[t_] := -240.0 +
(23852 × (0.95 / (0.95 + (100.0 - 0.95) × (Exp[-(0.199 × (t - 3.05)]))))))
Table[{t, SetPrecision[test[t], 3]}, {t, 0, 60, 2}]
```

```
Out[54]= {{0, -116.}, {2, -55.8}, {4, 33.2}, {6, 164.}, {8, 357.}, {10, 639.},
{12, 1.04 × 103}, {14, 1.62 × 103}, {16, 2.43 × 103}, {18, 3.53 × 103}, {20, 4.97 × 103},
{22, 6.77 × 103}, {24, 8.89 × 103}, {26, 1.12 × 104}, {28, 1.36 × 104}, {30, 1.58 × 104},
{32, 1.77 × 104}, {34, 1.93 × 104}, {36, 2.05 × 104}, {38, 2.15 × 104}, {40, 2.21 × 104},
{42, 2.26 × 104}, {44, 2.29 × 104}, {46, 2.31 × 104}, {48, 2.33 × 104}, {50, 2.34 × 104},
{52, 2.35 × 104}, {54, 2.35 × 104}, {56, 2.35 × 104}, {58, 2.36 × 104}, {60, 2.36 × 104}}
```

```
In[55]:= TstE1L = ListPlot[Table[{t, test[t]}, {t, 0, 60, 2}],  
PlotRange -> {{0, 60}, {0, 25 000}}, Frame -> True,  
GridLines -> Automatic, ImageSize -> {575, 366}, Joined -> False,  
Mesh -> All, PlotStyle -> {RGBColor["Blue"], AbsolutePointSize[9]},  
AspectRatio -> 2 ÷  $\pi$ , FrameLabel -> {Style["Time (days)", Bold, Blue],  
Style["Cases (thousands)", Bold, Blue]},  
LabelStyle -> Directive[Blue, Large]]
```

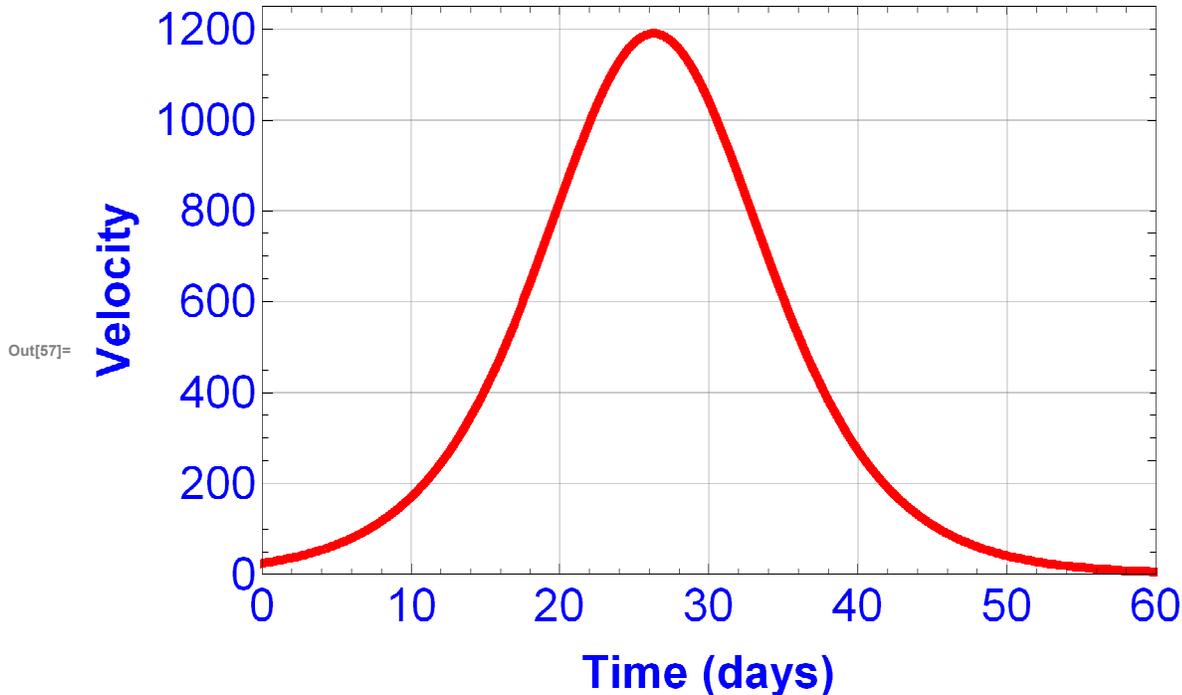


In[56]:= Show[TstE1L, dtplt1a]



In[57]:= **FirstDrv =**

```
Plot[{KinetE1L'[t]}, {t, 0, 60}, PlotRange → {{0, 60}, {0.0, 1250.0}},
Frame → True, GridLines → Automatic,
ImageSize → {575, 366}, PlotStyle → {Red, Thickness[0.01]},
AspectRatio → 2 ÷ π, FrameLabel →
{Style["Time (days)", Bold, Blue], Style["Velocity", Bold, Blue]},
LabelStyle → Directive[Blue, Large]]
```



In[58]:= **FindMaximum[KinetE1L'[t], {t, 25}]**

Out[58]= {1190.27, {t → 26.3037}}

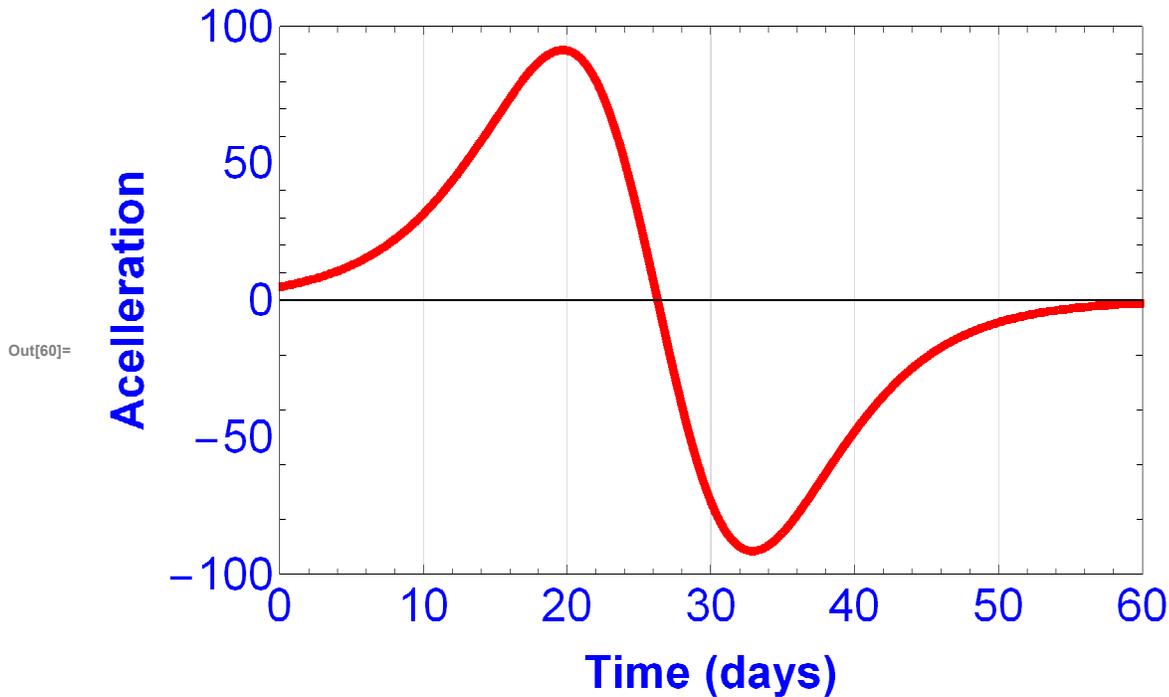
In[59]:= **FindMinimum[KinetE1L'[t], {t, 1000}]**

**FindMinimum:** Encountered a gradient that is effectively zero. The result returned may not be a minimum; it may be a maximum or a saddle point.

Out[59]= {1.86569 × 10<sup>-81</sup>, {t → 1000.}}

In[60]:= **SecondDrv =**

**Plot[{KinetE1L''[t]}, {t, 0, 60}, PlotRange → {{0, 60}, {-100.0, 100.0}},  
 Frame → True, GridLines → Automatic, ImageSize → {575, 366},  
 PlotStyle → {Red, Thickness[0.01]}, AspectRatio → 2 ÷  $\pi$ ,  
 FrameLabel → {Style["Time (days)", Bold, Blue],  
 Style["Accleration", Bold, Blue]},  
 LabelStyle → Directive[Blue, Large]]**



In[61]:= **FindMaximum[KinetE1L''[t], {t, 20}]**

Out[61]= {91.4454, {t → 19.7059}}

In[62]:= **FindMinimum[KinetE1L''[t], {t, 30}]**

Out[62]= {-91.4454, {t → 32.9015}}